

1 Flippin' Coins

Suppose we have a biased coin, with outcomes H and T , with probability of heads $\mathbb{P}[H] = 3/4$ and probability of tails $\mathbb{P}[T] = 1/4$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

- (a) What is the *sample space* for our experiment?
- (b) Which of the following are examples of *events*? Select all that apply.
- $\{(H, H, T), (H, H), (T)\}$
 - $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
 - $\{(T, T, T)\}$
 - $\{(T, T, T), (H, H, H)\}$
 - $\{(T, H, T), (H, H, T)\}$
- (c) What is the complement of the event $\{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, T, T)\}$?
- (d) Let A be the event that our outcome has 0 heads. Let B be the event that our outcome has exactly 2 heads. What is $A \cup B$?
- (e) What is the probability of the outcome (H, H, T) ?
- (f) What is the probability of the event that our outcome has exactly two heads?

2 Venn Diagram

Out of 1000 computer science students, 400 belong to a club (and may work part time), 500 work part time (and may belong to a club), and 50 belong to a club and work part time.

- (a) Suppose we choose a student uniformly at random. Let C be the event that the student belongs to a club and P the event that the student works part time. Draw a picture of the sample space Ω and the events C and P .

- (b) What is the probability that the student belongs to a club?
- (c) What is the probability that the student works part time?
- (d) What is the probability that the student belongs to a club AND works part time?
- (e) What is the probability that the student belongs to a club OR works part time?

3 Probability Practice

- (a) If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?

- (b) A message source M of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0, 1, 2\}$, and all such words are equally probable. What is the probability that M produces a word that looks like a byte (*i.e.*, no appearance of '2')?

- (c) If five numbers are selected at random from the set $\{1, 2, 3, \dots, 20\}$, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)

4 Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, *i.e.*, a birthday that has already been recorded. (Assume there are 365 days in a year.)

- (a) What is the probability that after the first 3 people's birthdays are recorded, no match has occurred (*i.e.* each person has a unique birthday)?
- (b) What is the probability that the first 3 people all share the same birthday?
- (c) What is the probability that it takes more than 20 people for a match to occur?
- (d) What is the probability that it takes exactly 20 people for a match to occur?
- (e) Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?