1 Clothes and Stuff

(a) Say we've decided to do the whole capsule wardrobe thing and we now have only 5 different items of clothing that we wear (jeans, tees, shoes, jackets, and floppy hats, etc.). We have 3 variations on each of the items, and we wear one of each item every day. How many different outfits can we make?

(b) It turns out 3 floppy hats really isn't enough of a selection, so we've bought 11 more, and we now have 14 floppy hats. Now how many outfits can we make?

(c) If we own k different items of clothing, with n_1 varitions of the first item, n_2 variations of the second, n_3 of the third, and so on, how many outfits can we make?

(d) We love our floppy hats so much that we've decided to also use them as wall art, so we're picking 4 of our 14 hats to hang in a row on the wall. How many such arrangements could we make? (Order matters.)

(e) Ok, now we're packing for vacation to Iceland, and we only have space for 4 of our 14 floppy hats. How many sets of 4 could we bring? (Yeah, yeah, we knew you were going to use that notation. Now tell us the number as a function of d, your answer from the previous part.)

(f) Ok, turns out the check-in person for our flight to Iceland is being *very* unreasonable about the luggage weight restrictions, and we're going to have to leave some hats behind. Despite our best intentions, and having packed only 4 hats, we actually bought 18 additional floppy hats at the airport (6 in burgundy, 6 in forest green, and 6 in classic black). We'll keep our 4 hats that we brought from home, but we'll have to return all but 6 of the airport hats. How many color configurations can there be for the 6 airport hats that we keep?

2 Fruits

Suppose you want to buy n fruits, and you can buy 0 or more of any type. In how how many ways can you do that if:

(a) There are apples and oranges at the market.

(b) There are apples, oranges, and bananas at the market.

(c) There are k kinds of fruits at the market.

3 Combinatorial Proof I

Prove $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

4 Teams and Leaders

Prove the following identities using a combinatorial proof.

1. $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

2. $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$