

1 Badminton Ranking

A team of n ($n \geq 2$) badminton players held a tournament, where every person plays with every other person exactly once, and there are no ties. Prove by induction that after the tournament, we can arrange the n players in a sequence, so that every player in the sequence has won against the person immediately to the right of him.

2 Seating Arrangement

N people have come to watch a play and were given a row with exactly N consecutive seats. They have decided on the following seating arrangement. After the first person sits down, the next person has to sit next to the first. The third sits next to one of the first two and so on until all N are seated. In other words, no person can take a seat that separates him/her from at least one other person. How many different ways can this be accomplished? Note that the first person can choose any of the N seats. [*Hint*: Use induction.]

3 Well-Ordered Grid

Consider an infinite sheet of graph paper such that each square contains a natural number. Suppose that the number in each square is equal to the average of the numbers in the four neighboring squares.

- By the Well-Ordering Principle, there must be some smallest number in the grid (call it n). Prove that for any square containing n , the four squares adjacent to it must also contain n .
- Prove that each square in the infinite grid contains the same number.

4 Elephant Mosquito Paradox

Claim: The weight of an elephant equals the weight of a mosquito.

Proof: Let x be the weight of an elephant, and y that of a mosquito. Call the sum of the two weights $2v$, so that

$$x + y = 2v.$$

From this equation we can obtain two more.

$$x - 2v = -y, x = -y + 2v$$

Multiplying those together, we get

$$x^2 - 2vx = y^2 - 2vy.$$

Add v^2 to both sides.

$$\begin{aligned}x^2 - 2vx + v^2 &= y^2 - 2vy + v^2 \\(x - v)^2 &= (y - v)^2\end{aligned}$$

Taking square roots, we get

$$x - v = y - v.$$

From this we conclude: $x = y$. That is, the elephant's weight (x) equals the mosquito's weight (y). Q.E.D. What is wrong here? You only need to find one wrong step, but identify all the wrong steps if you find more than one.