CS 70 Discrete Mathematics and Probability Theory
Summer 2017 Lu, Moulos, and Tang

DIS 1A

1 Set Operations

- $\mathbb{R}$, the set of real numbers
- $\mathbb{Q}$, the set of rational numbers: $\{a/b : a, b \in \mathbb{Z} \wedge b \neq 0\}$
- $\mathbb{Z}$, the set of integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- $\mathbb{N}$, the set of natural numbers: $\{0, 1, 2, 3, \ldots\}$

(a) Given a set $A = \{1, 2, 3, 4\}$, what is $\mathscr{P}(A)$ (Power Set)?

(b) Given a generic set B, how do you describe $\mathscr{P}(B)$ using set comprehension notation? (Set Comprehension is $\{x \mid x \in A\}$.)

(c) What is $\mathbb{R} \cap \mathscr{P}(A)$?

(d) What is $\mathbb{R} \cap \mathbb{Z}$?

(e) What is $\mathbb{N} \cup \mathbb{Q}$?

(f) What is $\mathbb{R} \setminus \mathbb{Q}$?

(g) If $S \subseteq T$, what is $S \setminus T$?

2 Writing in Propositional Logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

(a) The square of a nonzero integer is positive.

(b) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.

(d) For any two distinct real numbers, we can find a rational number in between them.

3 Implication

Which of the following implications are true? Give a counterexample for each false assertion.

(a) $\forall x, \forall y, P(x,y) \implies \forall y, \forall x, P(x,y)$.

(b) $\exists x, \exists y, P(x,y) \implies \exists y, \exists x, P(x,y)$.

(c) $\forall x, \exists y, P(x,y) \implies \exists y, \forall x, P(x,y)$.

(d) $\exists x, \forall y, P(x,y) \implies \forall y, \exists x, P(x,y)$.

4 Necessary and Sufficient Conditions

(a) Given implication $A \implies B$, A is a __________ condition for B.

(b) Given implication $\neg A \implies \neg B$, A is a __________ condition for B.

(c) Given implication $\neg B \implies \neg A$, A is a __________ condition for B.

(d) Given implication $B \implies A$, A is a __________ condition for B.